Multidisciplinary Optimization Group

OpenPulse - Open source code for numerical modelling of low-frequency acoustically induced vibration in gas pipeline systems

Theory Reference A: Acoustic gas pulsation module
V1.0

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1 Introduction

The acoustic gas pulsation module consists of an interface that allows to simulates 1D-plane wave propagation problems in acoustic fluids for piping. Mathematically, the motor behaviour of the module interface is designed to solve the 1D-wave equation in the plane wave mode frequency range using an analytic solution in the FETM formulation. Physically, an acoustic gas pulsation problem is being solved. The module gives the possibility to do a time harmonic analysis of the acoustic fluid.

The time harmonic analysis enables to solve pressure field inside the piping under time harmonic pressure and volume velocity sources as well as nodal impedances BC's. This is useful to model excitation sources as compressors, pumps and different piping elements as plate orifices, pressure vessels, valves and acoustic filters. The generated post-processing from this analyses is one of the first steps into the acoustic induced vibration analysis and allows to identify critics piping frequencies for gas pulsation phenomena.

2 Acoustic gas pulsation

Reciprocating/positive displacement pumps and compressors generate pressure fluctuations in the process acoustic fluid simply by virtue of the way in which they operate [11].

The gas (acoustic fluid) pressure fluctuation produced by this kind of machine can be intended as an incident pressure wave coming from an external pulsation source that is partially reflected and transmitted at every piping element such as orifice plates, valves, any throttling elements, reducers, expanders, etc [10].

Since these pressure fluctuations can reach amplitudes over 20 times higher than the dynamic pressure in main pipe [10] the structural vibration behavior of pipeline systems can be strongly affected by the response of the acoustic domain represented by the gas being transported through the pipes. These flow-related vibration phenomena are generally known as FIV. When flow-induced noise is present, the term FIVN is used. The term FIV became popular after [9] where was used in his book's title and probably for the first time, FIV phenomena were classified based on the two basics flow types: steady flow induced and unsteady flow related [1].

Particularly, deal with vibrations induced by compressor gas pulsation's is known as AIV. This vibration mechanism occurs in many industrial plants and is considered an important problem in industry because is a commonly cause of failure that obstructs smooth plan operations and in serious cases can lead to significant maintenance and repair costs and costly losses in productivity.

The AIV is a vibration mechanism which responds to an unsteady fluid flow and to a pulsating flow field [1]. Dealing with AIV means that the general random frequency characteristic of the fluid flow in the piping have a particular frequency component (and harmonics if broadband excitation exist) that becomes dominant when the interaction between the acoustic fluid and the compressor (intermittent suction/discharge compressor flow, Figure 1) occurs, generating pressure pulsations (acoustic fluid oscillations).

Reciprocating/positive displacement pumps and compressors can cause these pressure pulsations which can contain many harmonic components of the rotational speed (generally less than 100 Hz). Also centrifugal compressors can generate tonal pressure pulsations at low flow conditions and sub-synchronous tonal pressure component (10 to 80 % of rotor speed) caused by rotating stall. Generally, the pressure pulsation is to weak to cause any problem by it self, but if a coincidence with any pressure field and structural piping natural frequency occurs (Figure 1), the pressure pulsation can be largely amplified [2] and hence shaking forces [11].

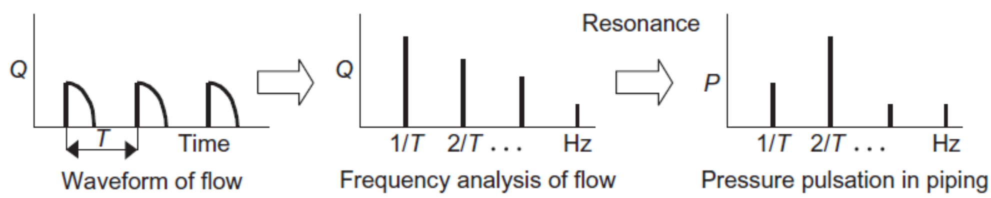

Figure 1: Suction/discharge flow and pressure pulsation in reciprocating compressors. Adapted from [2]

As the AIV phenomena has 3D complexity in nature, over 40 years models of pressure pulsation transmission of piping elements have been developed. The motivation is, in general, to have the overall transmission characteristics of an element represented by a simple but accurate enough 1D plane wave mode element model. These models are based on the TMM approach and in the following sections the necessary and essential theoretical background to obtain the TM of a 1D-straight uniform duct element as well as the procedures to model complex piping systems will be described.

3 Basic theoretical background of fluid analysis

For the fluid analysis two principal matrix methods will be analyzed. The first of them, the well known TMM which have typical applications in pipe systems for the study of wave propagation. The other method is the MMM a well known technique which enables to solve more complex pipe systems than the TMM.

The TMM allows to connect the field variables pressure p and volume velocity q from the inlet to the outlet trough the TM with a mixed nodal vector formulation. By contrary the MMM allows to connect the field variables pressure p and volume velocity q from the inlet to the outlet trough the MM with a non-mixed nodal vector formulation which is more suitable to couple with FEM. From the TM is possible to arrive to the MM and vice versa.

The main goal of the acoustic field analysis is to characterize the acoustic pressure for the plane wave frequency band, in all the points of a system under study and carry the information to the FEM structural analysis as and external load under certain coupling conditions.

From the acoustic gas pulsation phenomena view a particular analysis at certain characteristics frequencies generated by the relation between the excitation source, the resultant pressure field and structural response is of great interest .

In the following sections the TM formulation for the 1D hard-walled straight uniform duct element, the MM formulation, it assembly process and BC's insertion for the MM under the name FETM, will be briefly presented.

3.1 Transfer Matrix Method

3.1.1 Introduction

The TMM [14, 32, 5, 2, 10] is an analytical method which has been used to solve a variety of linear and non-linear dynamic or static problems in engineering especially for chain-type and topological systems (i.e. pipes and its related elements).

The TMM is a powerful mathematical technique to build discretized models for performing analytical and numerical studies of the plane wave acoustics in pipe systems. This method is particularly efficient to deal with plane acoustical waves in tubular circuits, because of the following aspects of the formulation [5]:

- Two scalar fields p and q are sufficient to describe the waves.
- Use of exact analytical wave solutions for low order modes propagation (plane waves)
- Easy representation of constant or varying cross-sectional ducts (i.e. horn geometries)
- Only a few types of transfer matrices are necessary to assembly for discretized tubular models.
- The degree of accuracy is independent of the range of wavelengths explored (within the limits of validity of the plane wave approximation)
- Certain implementations and changes in BC's (i.e inner or radiation nodal impedances, position and kind of nodal sources) and extension of the transfer matrix method to dissipative problems can be done maintaining the elementary formulation.

In the following section the TM for the 1D hard-walled straight uniform duct element will be presented.

3.1.2 Transfer matrix of a dampingless uniform 1D tube element

Considering an uniform tube length as shown in Figure 2, fluid properties like mean density ρ_f and speed of sound c_f are assumed to be uniform inside the tube. The volume velocity and pressure at the inlet of the element are denoted q_1 and p_1, while q_2 and p_2 denote the corresponding quantities at the outlet. Z represents the acoustical impedance of the tube element. The objective is to find a matrix equation which express the volume velocity $q(x,k)$ and the pressure $p(x,k)$ at any point x inside the tube element at the wave number (frequency) $k = \omega/c_f$, in terms of they values at the inlet. Accordingly, the tube element can be represented as a matrix linear system with two inputs and two outputs as follows

$$\begin{bmatrix} p_2 \\ q_2 \end{bmatrix} = \mathbf{T} \begin{bmatrix} p_1 \\ q_1 \end{bmatrix} \tag{1}$$

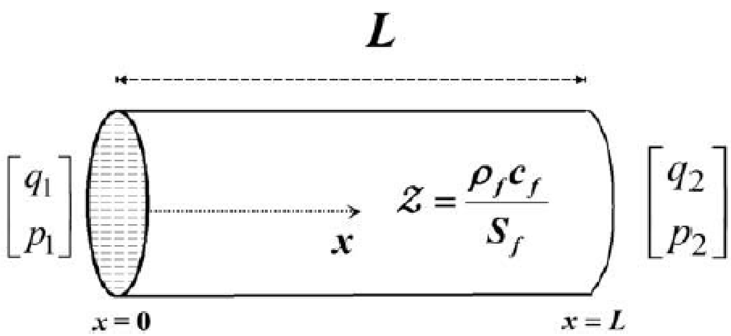

Figure 2: Uniform tube element. [5]

where $\mathbf{T}$ is the transfer matrix for the uniform tube element wich has four elements inside as follows

$$[\mathbf{T}]_{ij} = \begin{bmatrix} T_{11}(x,k) & T_{12}(x,k) \\ T_{21}(x,k) & T_{22}(x,k) \end{bmatrix} \tag{2}$$

where to obtain each of that elements we need to solve the wave equation under the assumption of plane waves. By definition , the volume offered to the fluid in a tube is characterized by one dimension x, which is much larger than the others two (the transverse dimensions). As a consequence, physically, for a fluid oscillation at a given frequency ω the compressibility parameter is much larger in the longitudinal direction than the others. Moreover if ω is small enough the compressibility parameter can be taken in account just in the greater direction. So, the idea is as larger is the wavelength λ, better works the plane wave assumption because the transverse dimensions smaller each time. Then, we can assume constant pressure in the hole tube cross-section and establish a frequency range validity for this assumption as

$$\frac{\lambda}{r_i} = \frac{c_f}{\omega r_i} >> 1 \tag{3}$$

where r_i is the inner duct radius. For this simple case, the homogeneous and non dissipative wave equation is

$$p_{,xx} - \frac{1}{c_f^2} p_{,tt} = 0 \tag{4}$$

then the 1D solution is written in terms of traveling waves for the pressure as

$$p(x,t) = \left(Ae^{-ikx} + Be^{ikx}\right) e^{i\omega t} \tag{5}$$

and with the aid of Euler's equation

$$q(x,t) = \frac{S_f}{\rho_f c_f} \left(Ae^{-ikx} - Be^{ikx}\right) e^{i\omega t} \tag{6}$$

where q is the interior nodal volume velocity and is equal to $q_n = S_{element_n} u_n$. From the mathematical standpoint, the two constants of integration A and B appearing in the general solution of the local equation 4 are univocally defined if both the pressure and the volume velocity are specified at the inlet of the tube element.

Evaluating equations 5 and 6 in $x = 0$ and $x = L$ the constants A and B are eliminated and we obtain the components of the transfer matrix as

$$[\mathbf{T}]_{ij} = \begin{bmatrix} cos(kx) & -i\, Z_f\, sin(kx) \\ -\frac{i}{Z_f}\, sin(kx) & cos(kx) \end{bmatrix} \tag{7}$$

and we can complete now the matrix system (presented in equation 1) wich involves the connection between the two-port nodes p and q at the input and at the output of the straight tube element with the transfer matrix $\mathbf{T}$

$$\begin{bmatrix} p_2 \\ q_2 \end{bmatrix} = \begin{bmatrix} cos(kx) & -i\, Z_f\, sin(kx) \\ -\frac{i}{Z_f}\, sin(kx) & cos(kx) \end{bmatrix} \begin{bmatrix} p_1 \\ q_1 \end{bmatrix} \tag{8}$$

where $Z_f = \rho_f c_f / S_f$ is the acoustic fluid impedance. Observing that the matrix 7 is not symmetrical (self-adjoint) cause by the choice made in the definition of the input and output vectors which mix kinematics and stress variables (mixed formulation) [5].

3.1.3 Transfer matrix of a damped uniform 1D tube element

For the damped case, the same dampingless TM structure is used due to the choice of the damping insertion trough a (dissipative and dispersive)complex wavenumber k_c, which its definition changes depending on the damping model.

Inside the OpenPulse Acoustic gas pulsation module five possibilities of acoustic elements are available to choose, each one with a different damping degree and useful for a wide range of applications. These five damping models and, as a consequence, five different acoustic elements are listed below and will be described in the follow subsections.

1. Dampingless
2. Hysteretic
3. Helmholtz-Kirchoff wall-attenuation coefficient
4. Low Reduced Frequency Equivalent Fluid thermoviscous (LRF-EF)
5. Low Reduced Frequency full thermoviscous (LRF-full)

The first four listed options use the same TM structure and are based on the dampingless TM deduction. As was mentioned, each damping model is reflected in the TM trough a complex wave number insertion. The complete mixed TM system with q, p nodal vectors for this first four listed damping models is

$$\begin{bmatrix} p_2 \\ q_2 \end{bmatrix} = \begin{bmatrix} cos(k_c\, x) & -i\, Z_f\, sin(k_c\, x) \\ -\frac{i}{Z_{f,c}}\, sin(k_c\, x) & cos(k_c\, x) \end{bmatrix} \begin{bmatrix} p_1 \\ q_1 \end{bmatrix} \tag{9}$$

where $k_c = k_1 - ik_2$ and $Z_{f,c} = \rho_{f,c} c_{f,c} / |S_f$. Being $k_1 = k = \omega / c_{f,0}$ and k_2 the real and imaginary part of the complex wave number and $\rho_{f,c}$ and $c_{f,c}$ are the complex fluid density and the complex fluid speed of sound. The definition and values of k_2, $\rho_{f,c}$ and $c_{f,c}$ depends on each damping model and will be described in the following subsections.

The last damping model (LRF-full) has a different TM because its deduction, roughly speaking, is not based on Equation 4 [33]. Then, the complete mixed TM system with q, p nodal vectors for this damping models is

$$\begin{bmatrix} p_2 \\ q_2 \end{bmatrix} = \begin{bmatrix} cosh(k_c\, x) & -i\frac{Z_{f,c}}{S\Gamma<\alpha>_r}\, sinh(k_c\, x) \\ i\frac{S\Gamma<\alpha>_r}{Z_{f,c}}\, sinh(k_c\, x) & cosh(k_c\, x) \end{bmatrix} \begin{bmatrix} p_1 \\ q_1 \end{bmatrix} \tag{10}$$

where $< \alpha >_r = 1 - \big(2J_1\big((\sqrt{2}/2)s(1-i)\big)/J_0\big(i^{3/2}(\sqrt{2}/2)s(1-i)\big)\big)$.

3.1.4 Hysteretic damping

The hysteretic damping model is presented in OpenPulse as a user-defined attenuation fluid property. Its value is typically obtained of experimental data and could be expressed as a part of the fluid real valued wavenumber k. Also, the hysteretic attenuation coefficient (also called material's loss coefficient) η could be frequency-dependent, $\eta(f)$, or frequency-independent, η. In OpenPulse, a plane wave ansatz (omitting time dependence), for example, in (positive)x direction considering hysteretic damping is calculated as

$$k_c = k(1 - i\eta) \tag{11}$$

$$p(x) = e^{-i\,k_c x} = e^{-i\,kx}\, e^{-\eta x} \tag{12}$$

where η is dimensionless and frequency-independent. In equation 11, the (loss)imaginary part of k_c, k_2, was expressed as a part of the (lossless)real wavenumber $k_1 = k$ governed by η. Also, it is important to note that for the propagation's positive direction and the chosen sign for η in equation 11, a wave with spatial exponential decay governed by the hysteretic attenuation coefficient appears. For a plane wave ansatz (omitting time dependence), in negative x direction

$$p(x) = e^{i\,k_c x} = e^{i\,kx}\, e^{\eta x} \tag{13}$$

where the sign change in the plane wave propagation direction affects the wave spatial exponential decay governed by η. For this damping model, the fluid complex sound speed is defined as

$$c_{f,c} = \frac{c_{f,0}}{1 - i\eta} \tag{14}$$

and for the complex fluid density

$$\rho_{f,c} = \rho_{f,0}(1 - i\eta)^2 \tag{15}$$

3.1.5 Helmholtz-Kirchoff wall-attenuation coefficient

When a plane-wave ansatz is being propagated trough an acoustic fluid in a tube, it suffer an attenuation. This total attenuation is composed by the attenuation or absorption in the acoustic fluid and by the dissipation at the tube's wall. In order to obtain a measure of the absorption in the material, the dissipation due to wall effects must be determined independently, analytically o experimentally.

To solve this problem analytically some approximation needs to be done as to assume that viscous and thermal effects can be calculated separately and then summed to obtain the total-wall-attenuation. Also, was considered that no other energy-loss mechanism exist in the tube.

Finally, considering small dissipation it is necessary that the volume affected by viscous and thermal effects must be small compared to the total acoustic fluid's volume in the tube. Therefore, this leads to the small dissipation condition [31]

$$\sqrt{\frac{2\nu_{f,0}}{\omega}} << r \tag{16}$$

$$\sqrt{\frac{2\kappa_{f,0}}{\omega}} << r \tag{17}$$

$$\sqrt{\frac{\omega\nu_{f0}}{c_{f,0}^2}} << 1 \tag{18}$$

which reflects (into other things) an order magnitude relation between the tube radius and thermal and viscous penetration depth (acoustic fluid volume affected by the walls). Also, as was assumed plane-wave propagation

$$r < \lambda \tag{19}$$

denoting that the radius of the tube cannot be arbitrarily large [31].

The Helmholtz-Kirchhoff (total)wall-attenuation coefficient can be obtained as the sum of the viscous and thermal dissipation's coefficients and being included inside the complex wavenumber as [31]

$$k_c = k - i\frac{\sqrt{\omega\nu_f}}{\sqrt{2}c_{f,0}r}(1 + \frac{\gamma - 1}{\sqrt{Pr}}) \tag{20}$$

where Pr is the Prandtl number, ν_f is the undisturbed fluid kinematic viscosity and $\gamma = Cp/Cv$ the ratio of specific heats. Then the fluid complex sound speed for the Helmholtz-Kirchoff damping is defined as

$$c_{f,c} = c_{f,0}\left[1 - i\frac{\sqrt{\nu_f}}{\sqrt{2\omega}\, c_{f,0}\, r}\left(1 + \frac{\gamma - 1}{\sqrt{Pr}}\right)\right] \tag{21}$$

and for the complex fluid density

$$\rho_{f,c} = \frac{\rho_{f,0}}{\left[1 - i\frac{\sqrt{\nu_{f,0}}}{\sqrt{2\omega}\, c_{f,0}\, r}\left(1 + \frac{\gamma-1}{\sqrt{Pr}}\right)\right]^2} \tag{22}$$

Is good to remember for the OpenPulse user that as was expressed in equations 16, the equation 20 works well for r relatively large and ω relatively small (wide-tube, low-frequency wall attenuation coefficient approximation). As the frequency increases, the agreement between theory and experiments is not good. Some other effects non-included into the presented theoretical result are [31]

- Absoprtion at the reflected end in closed tubes
- Nonlinearities of the sound field
- Wall-surface roughness
- (Local)Sound radiation trough imperfect seals
- Rotational relaxation expressed trough expansive viscocity μ_ν
- Vibrational relaxation

A detailed theoretical development of this wall-attenuation-coefficient and its physical behaviour can be found in chapter 6 of [31].

3.1.6 LRF-EF

The material in this subsubsection is largely based on the Ph.D. thesis [23].

The Low reduced frequency (LRF) model (name coined by Tijdeman [33]) describes viscothermal acoustics in tubes and layers that have a cross section smaller than the acoustic wavelength (inside the plane wave frequency range). A LRF model can be derived from analytic solutions solving the full linearized Navier-Stokes set of PDES listed below

$$\begin{aligned} & i\omega\rho_0\mathbf{v} - \nabla\cdot\sigma = f \\ & i\omega\rho_0 C_p T + \nabla\cdot\mathbf{q} - i\omega p = Q \\ & \nabla\cdot\mathbf{v} - i\omega\frac{T}{T_0} + i\omega\frac{p}{p_0} = 0 \end{aligned} \tag{23}$$

where the chosen degrees of freedom were, the velocity vector $\mathbf{v}$, the temperature T and the pressure p. The set of equations 23 are used to represent full detailed thermoviscous acoustic models. But, using FEM to solve these equations could be computationally costly, principally when the frequency and the number of boundary layers rises in the model. Also when analyzing long piping systems. To sort a high computationally cost model, it is possible to add the losses associated with the boundary layer onto the bulk of the fluid, this is, an equivalent fluid model (EF model).

For this, the set of equations 23 are rewritten in a dimensionless set of equations

$$\begin{aligned} & \tilde{v} + \tilde{k}_v^{-2}\tilde{\xi}\tilde{\nabla}(\tilde{\nabla}\cdot\tilde{v}) + \tilde{k}_v^{-2}\tilde{\nabla}\tilde{v} = -\frac{\tilde{\nabla}\tilde{p}}{i\gamma} \\ & \tilde{T} + \tilde{k}_h^{-2}\tilde{\nabla}\tilde{T} = \frac{\gamma-1}{\gamma}\tilde{p} \\ & \tilde{\nabla}\cdot\tilde{v} - i(\tilde{T} - \tilde{p}) = 0 \end{aligned} \tag{24}$$

where the source terms $\mathbf{f}$ and Q were set to zero and the dimensionless variables were indicated with ˜ accent.

In the bulk of the fluid, for an acoustic plane wave, the dimensionless chosen degree of freedom $\tilde{\mathbf{v}}$, $\tilde{T}$, $\tilde{p}$ and their dimensionless derivatives are of equal order. This results in neglecting all terms containing $\tilde{k}_v^{-2}$ and $\tilde{k}_h^{-2}$ (second order small terms). In this way, it is possible to arrive to the non-dissipative homogeneous wave equation presented in equation 4.

Within the boundary layer, the temperature and the shear velocity rapidly change from the value in the bulk to zero at the boundary. Therefore, the temperature and velocity gradients are first order large that the variable itself and their Laplacians are second order large. The pressure and pressure gradient remain relatively smooth over the boundary.

Based on these comments, the term containing $\tilde{\xi}$ in equation 24 is first order small and can be neglected with respect to the pressure gradient resulting in the following final set of equations

$$\begin{aligned} & \tilde{v} + \tilde{k}_v^{-2}\tilde{\nabla}\tilde{v} = -\frac{\tilde{\nabla}\tilde{p}}{i\gamma} \\ & \tilde{T} + \tilde{k}_h^{-2}\tilde{\nabla}\tilde{T} = \frac{\gamma-1}{\gamma}\tilde{p} \\ & \tilde{\nabla}\cdot\tilde{v} - i(\tilde{T} - \tilde{p}) = 0 \end{aligned} \tag{25}$$

The set of equations 25 allow LRF approximate viscothermal analytical solutions for many geometries and can be applied in a wide range of engineering applications under different assumptions and constraints. The LRF models cover the range from fully isothermal conditions (very low frequencies or very narrow tubes) to large ducts where the boundary layer only represents a fraction of the duct size (wide tubes).

The four assumptions that lead to the LRF model are

1. The viscous and thermal wave numbers must be much larger than the acoustic wave number

$$\begin{aligned} \left|\frac{k_v}{k}\right| << 1 \\ \left|\frac{k_h}{k}\right| << 1 \end{aligned} \tag{26}$$

2. The cross section must be much smaller than the acoustic wavelength

$$He << 1 \tag{27}$$

3. The cross section should be uniform, or slowly varying, such that the cross section velocity components can be neglected

4. The length of the geometry in the propagation direction should be large compared to the boundary layer thickness

$$\begin{aligned} e_{size} > \delta_v \\ e_{size} > \delta_h \end{aligned} \tag{28}$$

The dimensionless variable $He = kr$ is the Helmholtz number and is commonly called in the literature as reduced frequency. Therefore, the name LRF is a statement of the second requirement that allows an approximation of uniform pressure over the cross section surface. The third requirement ensures that the velocity is mainly directed in the propagation directions. The fourth requirement is only necessary to make the viscothermal inlet effects negligible. Violation of the third and fourth requirements, in some cases, can be easily compensated by the application of acoustic end corrections.

After mentioning these assumptions the approximate viscothermal solutions obtained from solve approximately the two first equations in the set of equation 25 are presented. The approximate temperature solution can be written as

$$\tilde{T} = \Psi_t \frac{\gamma - 1}{\gamma} \tilde{p} \tag{29}$$

being Ψ_t a complex valued scalar (thermal)field which its value depends on the fluid geometry and the implemented BC's. In this case, for a rigid straight cylindrical tube with isothermal boundary conditions the (lumped)thermal field is

$$\Upsilon_t = \langle \Psi_t \rangle_{\partial\Omega} = \frac{1}{S}\int_{\partial\Omega} \Psi_t \, d\partial\Omega = \frac{1}{S}\int_{\partial\Omega} 1 - \frac{J_0(k_t R)}{J_0(k_t r)} \, d\partial\Omega = 1 - \frac{J_2(k_t r)}{J_0(k_t r)} \tag{30}$$

where $R = [0, r]$. On the other hand, the shear velocity approximate solution is

$$\tilde{v} = -\frac{\Psi_v \tilde{\nabla} \tilde{p}}{i\gamma} \tag{31}$$

being Ψ_v a complex valued scalar (viscous)field which also depends on the fluid geometry and BC's. In this case, for a rigid straight cylindrical tube with no slip boundary conditions the (lumped)viscous field is

$$\Upsilon_v = \langle \Psi_v \rangle_{\partial\Omega} = \frac{1}{S}\int_{\partial\Omega} \Psi_v \, d\partial\Omega = \frac{1}{S}\int_{\partial\Omega} 1 - \frac{J_0(k_v R)}{J_0(k_v r)} \, d\partial\Omega = 1 - \frac{J_2(k_v r)}{J_0(k_v r)} \tag{32}$$

Once the (lumped)geometry and material-dependant complex field functions Υ_t and Υ_v are known, the complex wavenumber can be expressed as

$$k_c = k\sqrt{\frac{\Upsilon_t'}{\Upsilon_v}}, \tag{33}$$

where $\Upsilon_t' = \gamma - (\gamma - 1)\Upsilon_t$ is the modified (cross-sectional)mean thermal field, Υ_v the (cross-sectional)mean value of the viscous field Ψ_v, γ the ratio of specific heats and $k = \omega/c_{f,0}$ the non-dispersive and non-dissipative acoustic fluid wavenumber.

On the other hand the fluid complex sound speed for the LRF-FE thermoviscous damping is defined as

$$c_{f,c} = \frac{c_{f,0}}{\sqrt{\frac{\Upsilon_t'}{\Upsilon_v}}} \tag{34}$$

and for the complex fluid density

$$\rho_{f,c} = \frac{\rho_{f,0}}{\Upsilon_v} \tag{35}$$

3.1.7 LRF-full

Acoustic's models can be grouped into three basic categories. The first type and simplest models (but very useful), the LRF model that assumes a constant pressure across the tube's cross-section. The second type incorporates a pressure gradient but neglecting some terms of equation 23 and the third type and most extensive model taken into account all terms in equation 23 [7].

In this subsection, another LRF model will be presented but not using the complex wavenumber as a way to introduce the viscothermal effects. In the LRF-EF model, the value of k_c is a complex function of the ratio between thermal and viscous fields, respectively (equation 33). The LRF-full model presented in this subsection uses the temperature, velocity (only axial) and density solutions from the LRF approximation. The denomination full is not related to the use of all the terms in the equation 23 but yes by the fact that the pressure and axial velocity LRF solution obtained by Tijdeman in [33] were used to build the TM and MM matrix (equations 10 and 51).

For the LRF solution in [33] assumes that

$$He << 1 \tag{36}$$

$$\frac{v}{u} << 1 \tag{37}$$

$$\frac{He}{s} << 1 \tag{38}$$

being v and u the radial and axial velocity respectively. Assuming no-slip and isothermal BC's at the tube's wall and an axial symmetric velocity profile also as BC and an harmonic pressure perturbation as

$$p = (Ae^{\Gamma kx} + Be^{-\Gamma kx})e^{i\omega t} \tag{39}$$

being

$$k_c = \Gamma k \tag{40}$$

where $\Gamma = \sqrt{J_0(i^{3/2}s)/J_2(i^{3/2}s)}\sqrt{\gamma/n}$ is the propagation constant, s the shear wave number (also referred to as the Stokes number) and n is the politropic constant.

The solution for the axial velocity is (omitting time dependence)

$$u = \frac{i\Gamma}{\gamma}\left[1 - \frac{J_0(i^{3/2}\eta s)}{J_0(i^{3/2}s)}\right](Ae^{\Gamma kx} - Be^{-\Gamma kx}) \tag{41}$$

being $\eta = r/R$ a dimensionless radial coordinate for the moving parameter r and the fixed value of the tube radius R (using Tijdeman's notation). The equations 39 and 41 are radial and axial dependent. Because OpenPulse Acoustic module works with a 1D TM and MM formulation, these equations were lumped to obtain the equations 10 and 51.

The fluid complex sound speed for the LRF-full thermoviscous damping model is defined as

$$c_{f,c} = \frac{c_{f,0}}{\Gamma}, \tag{42}$$

and for the complex fluid density

$$\rho_{f,c} = \rho_{f,0}(1 + \rho' e^{i\omega t}), \tag{43}$$

where ρ' is the amplitude of density perturbation [33] and just could be calculated after solving the system equation 51 because is pressure-dependent (compressible fluid model). As a consequence the definition of $Z_{f,c}$ in this damping model could be suggested as

$$Z_{f,c} = \rho_{f,0} c_{f,c}. \tag{44}$$

3.1.8 Womersley number

The Womersley number (Wo or sometimes α, [35]) is useful to relate length scales of an acoustic model. For example, when is being considered to compare the thickness of the thermal or viscous boundary layer to a physical dimension of the model (typically the tube radius r). The Wo number can be expressed as

$$Wo = \sqrt{\frac{\omega \rho_{f,0} r^2}{\mu_{f,0}}} = \frac{r\sqrt{2}}{\delta_v} = \frac{r}{\delta_t}\sqrt{\frac{2}{Pr}} \tag{45}$$

In the literature it is often possible to find expressions of the Wo number as a function of the Re, St and Stk numbers.

3.1.9 Some concluding thoughts on LRF-thermoviscous models application

- Solve only for thermoviscous acoustics where and when necessary. Investigate if the viscous and/or thermal boundary layer thicknesses are comparable to the geometrical scale. Remember that viscous and/or thermal boundary layer depend on frequency and geometry scales.

- Check your that material parameters related with losses are nonzero. Some of them could be:

  - Bulk viscocity, μ_B
  - Coefficient of thermal conduction, κ
  - Coefficient of isobaric thermal expantion, α_0
  - Coefficient of isothermal compressibility β_T.

- Check Wo number of your model. A rule of thumb could be if $Wo < 0.1$, the effect associated with viscous and thermal losses can be ignored and there is not necessary to choose the LRF-FE or LRF-full models.

3.2 Finite Element Transfer Matrix Method

3.2.1 Introduction

A combinations of matrix methods generally can be used to improve the computational cost, i.e. of FEM approaches, or for a quickly analysis of complex piping systems. The FETM [32], , sometimes called too SMM [2, 14, 15] or MMM [20] was one of the first methods to reduce the high computational cost from solving large matrix systems with high number of elements and was developed in the early 90s with the [14, 20, 34] and [32] works.

The transfer matrix system presented in equation 8 is based on a mixed formulation [5]. As can be seen in that equation, the nodal input and output vector have two different field variables, p and q (kinematic and stress variables), inside each one . That is, from a mathematical point of view we are dealing with scalar and vector field variables, solution of wave equations originated by fundamental field equations.

To relate the TM with a FEM matrix form a few algebraic operations needs to be done with the objective of arrange the variables of the mixed formulation into one not mixed (the same type of variables in each nodal vector), which is more suitable with the FEM matrixes. In structural mechanics this method is know as the FETM [32].

In the following section the MM for the 1D hard-walled straight uniform duct element it coupling process, characteristics and BC's will be presented.

3.2.2 Mobility Matrix of a dampingless uniform hard-walled straight 1D duct

The transformation of the mixed nodal vector TM system into a non-mixed was suggested by different authors [14, 32, 20, 12].

Taking the process suggested by [14] (one of the firsts in applied this proceeding for duct acoustics), directly we can use the TM system of Equation 8 and apply to the TM of Equation 7. Then the elementary $\mathbf{K_{A,e}}$ matrix for a straight tube with $\mathbf{q}$ and $\mathbf{p}$ as nodal variables is

$$\mathbf{K_{A,e}} = \begin{bmatrix} -i\,cot(kx)\,/\,Z_f & i\,/\,Z_f\,sin(kx) \\ i\,/\,Z_f\,sin(kx) & -i\,cot(kx)/\,Z_f \end{bmatrix} \tag{46}$$

where $\mathbf{K_{A,e}}$ is the elemental mobility matrix [20] and is clearly a symmetric matrix. This name probably is the most appropriated based on the mechanical/acoustic analogue force-velocity/pressure-volume velocity. The $\mathbf{K_{A,e}}$ matrix also is called the acoustic stiffness matrix [14] probably using the structural analogy stiffness-displacement-force, the Admittance Matrix too [12, 29] (electroacoustic analogy for the mobility, inverse of the mechanical impedance) and the four-pole matrix [34] due to the 4 node variables involved. In essence, the matrix $\mathbf{K_{A,e}}$ still depends on harmonically functions and on the fluid wave-number k, that it's mean, frequency dependent ($k = \omega/c$).

The combination between the TMM and the FEM can be classified as an hybrid method. This is because from a technique point of view the elementary matrixes are developed using the TMM formulation and then coupled (in the MM formulation) to form the global matrix system using a FEM based assembly process. From a mathematical point of view, the heart of this hybrid method consists in the quadratic or linear finite element form functions substitution by analytic field solutions of wave equation (in this case pressure and velocity inside a 1D straight constant section duct element) [27].

Finally the complete non-mixed MM system with q and p nodal vectors is

$$\begin{bmatrix} -i\,cot(kx)\,/\,Z_f & i\,/\,Z_f\,sin(kx) \\ i\,/\,Z_f\,sin(kx) & -i\,cot(kx)/\,Z_f \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \tag{47}$$

In the matrix system presented in Equation 47, the pressure and volume velocity nodal vectors are isolated (non-mixed formulation), this is quite similar to the classical finite element acoustic system according to the equation [27]

$$\left(\mathbf{H} + i\omega\mathbf{D} - \omega^2\,\mathbf{Q}\right)\,\mathbf{p} = \mathbf{q} \tag{48}$$

On LHS of equation 48 the mass $\mathbf{Q}$, stiffness $\mathbf{H}$, and damping $\mathbf{D}$ global matrixes ([18] convention) are condensed into a single matrix as a function of frequency called the mobility matrix $\mathbf{K_A}(\omega)$. In a global sense the system in Equation 47 can be written as

$$\mathbf{K_A}\,\mathbf{p} = \mathbf{q} \tag{49}$$

where $\mathbf{p}$ are the nodal pressures and $\mathbf{q}$ are the inner nodal volume velocities.

3.2.3 Mobility Matrix of a damped uniform hard-walled straight 1D duct

As was mentioned in Section 3.2.2, the TM is transformed to obtain the MM in the same way that the dampingless TM case. The first four damping models listed uses the same TM and, as a consequence, the same MM just changing the k_c definition depending on each damping model. Therefore, the complete non-mixed MM system for the the first four damping models is

$$\begin{bmatrix} -i\,cot(k_c\,x)\,/\,Z_{f,c} & i\,/\,Z_{f,c}\,sin(k_c\,x) \\ i\,/\,Z_{f,c}\,sin(k_c\,x) & -i\,cot(k_c\,x)/\,Z_{f,c} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \tag{50}$$

and the complete non-mixed MM system for the last damping model (LRF-full) is

$$\frac{i\,S\,\Gamma\,<\alpha>_r}{Z_{f,c}\,sinh(k_c\,x)} \begin{bmatrix} -\,cosh(k_c\,x) & 1 \\ 1 & -\,cosh(k_c\,x) \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \tag{51}$$

3.2.4 Assembly process for the Mobility Matrix

The global mobility matrix $\mathbf{K_A}$ is obtained doing the same procedure as proposed in the FEM. Some brief comments about the basics of this process will be done but for a complete understanding about this topic the lecturer is referred to a classic specialized bibliography as [19, 13, 21, 6].

Each acoustic 1D element have two nodes and each node 1 DOF (pressure). Also, the elements can have arbitrary length and cross-sectional area and at each node connection the continuity of the field variables p and q is guaranteed. Briefly, the general steps for the assembly process of the n $\mathbf{K_{A,e}}$ is

- Node indexing in a convenient way ($\mathbf{K_A}$ bandwidth dependant)
- Construction of the connectivity table
- Assembly of the n $\mathbf{K_{A,e}}$ according to the connectivity table.

3.2.5 Boundary Conditions

In the global system presented in Equation 49 the pressure field nodal vector $\mathbf{p}$ is presented as an unknown variable and must be solve. Before solve the system inverting the global mobility matrix $\mathbf{K_A}$ as indicated in equation 52 is necessary to apply all the corresponding BC's into the global matrix .

$$\mathbf{p} = \mathbf{K_A}^{-1}\,\mathbf{q} \tag{52}$$

Three general BC's can be applied in the FETM formulation

- Inner nodal volume velocity, q
- Nodal pressure, p
- Nodal acoustic impedance, Z

When one of these BC's is known is common to refer as prescribed BC's. The followings items briefly describe how each BC is applied.

- Volume velocity

  The simplest BC is a prescribed nodal volume velocity source $q_i = S_{e_n}\, u_i$ is prescribed on node $i = j$. If there are any other q volume velocity source at the $N-1$ remain interior nodes [14], the global matrix system for the piping becomes

$$[\mathbf{K_A}]_{ij} \begin{bmatrix} p_j \\ p_{j+1} \\ p_{j+2} \\ \vdots \\ p_{J-1} \\ p_J \end{bmatrix} = \begin{bmatrix} q_j \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}. \tag{53}$$

  when the value of q_j could be frequency dependant $q_j(\omega)$.

- Pressure

  If a nodal pressure is prescribed the classical process applied in FEM when a prescribed displacement occurs needs to be done as in [17]. The global system equation 49 in its final form becomes

$$\begin{bmatrix} 1 & 0 & \dots & 0 & 0 \\ 0 & K_A^{(i+1)(j+1)} & \dots & K_A^{(i+1)(J-1)} & K_A^{(i+1)J} \\ \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & K_A^{(I-1)(j+1)} & \dots & K_A^{(I-1)(J-1)} & K_A^{(I-1)J} \\ 0 & K_A^{I(j+1)} & \dots & K_A^{I(J-1)} & K_A^{IJ} \end{bmatrix} \begin{bmatrix} \overline{p}_j \\ p_{j+1} \\ \vdots \\ p_{J-1} \\ p_J \end{bmatrix} = \begin{bmatrix} p_j \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} - \overline{p}_j \begin{bmatrix} 0 \\ K_A^{(i+1)j} \\ \vdots \\ 0 \\ 0 \end{bmatrix} \tag{54}$$

  when the value of p_j could be frequency dependant $p_j(\omega)$.

- Impedance

  For the case when a node is connected to a system that has a known acoustic impedance Z the effect of this are added into appropriate (node) locations of the global mobility matrix [15] in the acoustic admittance form ($A = 1/Z$). If an acoustic radiation impedance A_{rad} wants to be added in the end of the tube, the matrix system of equation 49 becomes

$$\begin{bmatrix} 1 & 0 & \dots & 0 & 0 \\ 0 & K_A^{(i+1)(j+1)} & \dots & K_A^{(i+1)(J-1)} & K_A^{(i+1)J} \\ \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & K_A^{(I-1)(j+1)} & \dots & K_A^{(I-1)(J-1)} & K_A^{(I-1)J} \\ 0 & K_A^{I(j+1)} & \dots & K_A^{I(J-1)} & K_A^{IJ} + A_{rad} \end{bmatrix} \begin{bmatrix} \overline{p}_j \\ p_{j+1} \\ \vdots \\ p_{J-1} \\ p_J \end{bmatrix} = \begin{bmatrix} q_j \\ q_{j+1} \\ \vdots \\ q_{J-1} \\ 0 \end{bmatrix} \tag{55}$$

  where $A_{rad} = 1/Z_{rad}$.

From a user impedance input point of view, just the specific impedance (z) should be inserted in the BC's windows of acoustic gas pulsation module. Then, internally, the software makes the z_r transformation to Z_r dividing the value of the specific radiation impedance by the correspondent element area S_n and after an inversion the acoustic admittance A_r finally is obtained. This is done to avoid area and impedance type user's insertion errors.

However to avoid these errors, all the impedance BC's entry possibilities will have the correspondents dimensional units and a possibility to insert a measured acoustic or specific impedance varying with frequency, i.e with a *.txt* form, will be possible.

To clarify a little more the mean of different impedance types, the definitions proposed by [25] are presented

- Specific impedance = $z = p\,/\,u$ = pressure / normal particle velocity = $[Pa\,s/m]$
- Acoustic impedance = $Z = z\,/\,S = p\,/\,q$ = pressure / normal volume velocity = $[Pa\,s/m^3]$.
- Acoustic radiation impedance = $Z_r = z_r\,/\,S$ = pressure / normal volume velocity = $[Pa\,s/m^3]$
- Mechanical impedance = $Z_m = S\,z = S^2\,Z$ = force / particle velocity = $[N\,s/m]$

where S is the boundary surface or the cross-sectional element area which better represents the node where the impedance BC is applied and pressure and velocity in Z_r are evaluated at the open end of a tube. Volume velocity sometimes is referred as volume flow rate.

The following wave-guide end impedances exclusive for model pipe ends with absence of mean-flow are available in BC section of the Acoustic gas pulsation module

- Specific Anechoic impedance (fluid plane wave impedance) = $z_{end} = \rho_f\,c_f$
- Specific radiation unflanged pipe impedance, circular [4, 8]:

$$z_{unflanged} = \rho_f c_f \;(0,25\,(kr_i)^2 + i\;0,6133kr_i) \qquad \left[\tfrac{Pa\,s}{m}\right] \qquad ; \qquad kr_i < 0,5.$$

- Specific radiation unflanged pipe impedance, circular (normal incidence of plane waves, θ=0) [26]:

$$z_{unflanged} = \rho_f c_f \left(\frac{1+R}{1-R}\right) \qquad \left[\frac{Pa\;s}{m}\right] \qquad ; \qquad kr_i < 3,832. \tag{56}$$

$$R = |R|\,e^{2ikr_i\delta} \tag{57}$$

$$|R| = e^{(-kr_i)^2/2}\left[1 + \frac{(kr_i)^4}{6}\,ln\left((\gamma kr_i)^{-1} + \frac{19}{12}\right)\right] \qquad ; \qquad kr_i < 1. \tag{58}$$

$$|R| = \sqrt{\pi kr_i}\,e^{-kr_i}\left(1 + \frac{3}{32(kr_i)^2}\right) \qquad ; \qquad 1 < kr_i < 3,832. \tag{59}$$

$$\tag{60}$$

- Specific radiation flanged pipe impedance, circular [8]

$$z_{flanged} = \rho_f c_f \;\left(1 - \;\tfrac{2J_1(2kr_i)}{2kr_i} + i\;\tfrac{2H_1(2kr_i)}{2kr_i}\right) \qquad \left[\tfrac{Pa\,s}{m}\right].$$

where δ is the end correction [26], an interpolation function found by numerical integration, $\gamma = e^{0,5772}$, J_1 is the Bessel function of order 1, H_1 is the Struve function of order 1, the subscript f refers to the word fluid, r_i the inner ratio of the last tube element and k the wave number.

- Reciprocating compressor excitation
  - Waiting for new version.

3.3 Acoustic length correction

The use of 1D analytical acoustics models are very popular and useful for a wide range of application in engineering fields. However, generally exists certain limitations in their application as in the frequency range or in the geometry shape or in certain geometry aspect relations.

Generally, under the 0-order mode propagation (plane wave frequency range), simple geometries as plates, channels or circular or rectangular tubes, diameters or geometry lengths greater than viscous and/or thermal boundary layer, the 1D-acoustic analytical propagation models are in good agreement with 3D numerical or analytical wave propagation models but is typical to observe frequency shifts and magnitude differences when 1D and 3D responses are compared.

To avoid these differences exists the acoustic length correction (namely adopted for the case of pure acoustics) which its principally effect is to achieve a better prediction of local effects that are greatly represented by 3D models, even more when the physical phenomena has a 3D nature. Therefore, for complex pipes, when expansion chambers, side branches and loop geometries are present, a notable difference exists in the field variable's final prediction when the ALC was applied or not.

These differences can also become more important when applying boundary conditions at the end of a short acoustic tube element which its propagation equation was derived as a solution for waveguides of infinite length . These solution are really common in TPM development (i.e., TM, MM) causing an omission of higher order acoustic modes (thermal and viscous (evanescent)modes, the so-called inlet effects). However, applying the ALC to the waveguide with the smallest cross sectional dimensions, these higher order acoustic modes can be represented accurately [29].

All the acoustic elements available in the OpenPulse gas pulsation module were derived with solutions considering waveguides of infinite length and intended for low frequency applications. In general, the velocity, temperature and pressure profiles will not match at the interface of a geometry discontinuity (such as sudden expansion/contraction, side branch or loop geometries) between two duct acoustic elements (Figure 3). In fact, over the waveguide cross section close to that geometric discontinuity's interface, the pressure field could vary substantially [29]. This not-match is caused by the available acoustic formulations that were based on continuity of pressure and fluid flow continuity in an averaged sense only.

Most of the ALC expressions derived in the literature are unfortunately for inviscid adiabatic fluids and also frequency independent. This is, it was not taken into account viscous and thermal effects and the expression is not capable to reproduce a possible variation of the ALC into a frequency range. Certainly, most of the available solutions are intended for low frequency applications and certain conditions related to the importance of the viscothermal effects of the wave propagation (i.e., viscous effects are confined to boundary layers and could be modeled as an equivalent admittance [29, 28]).

To the author's knowledge (and in agreement with [29]) in general, no appropriate expressions are available in case viscous effects are significant (LRF-full thermoviscous models in the range $s<5$), but this case are out of the validity range of application of model 5), available in the OpenPulse gas pulsation module. Also, the author would like to note that , presumably, for the same geomtery, BC's, and frequency range and comparing with a dampingless model, the ALC

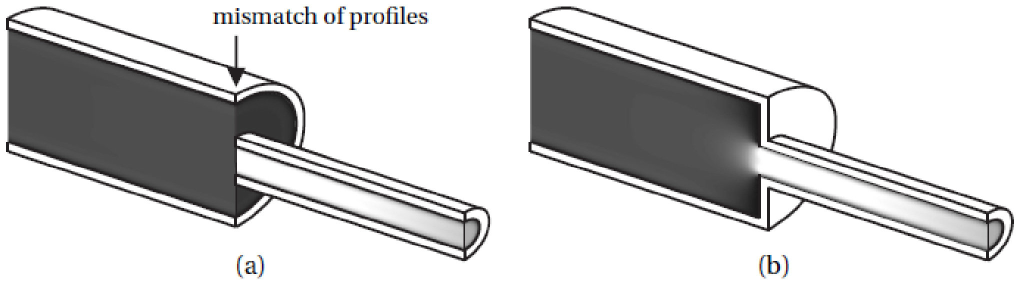

Figure 3: Velocity field mismatch at the interface of a sudden contraction/expansion geometric discontinuity. TM LRF solution (a)and FEM solution (b). Adopted from [29].

diminishes as more important are the thermoviscous effects. As a consequence, there are not available appropriate ALC expressions that account for viscous and thermal effects in the range s>5. Presumably Finally, as the literature suggest ([29]) and from various numerical test done by the author, the actual implemented ALC formulations can be used with relatively small error within the validity range of the available damping models in OpenPulse.

In the next subsections will be presented the used references for the firs instance application of the ALC for different kind of geometries discontinuities usually found in piping systems. As was mentioned in this brief introduction to the present topic, the ALC is not a simple subject to treat and their application in the cases when viscothermal effects are present at low frequencies is under continually improvement and study for the next OpenPulse Acoustic gas pulsation module versions.

3.3.1 Sudden expansions/contractions

The formulation suggested by Karal [24] is used for account the local effects in a sudden expansion or contraction. A practical engineering expression of the original equation in [24] can be found in [30].

3.3.2 Side branches

The formulation proposed by Ji [22] was used. Other formulation which includes experimentation and a theoretical model accounting losses due to viscosity and heat conduction can be found in [16].

3.3.3 Loop

A new formulation for loop geometries is under study. Temporally and after a few numerical tests, OpenPulse gas pulsation acoustic module is using the same length correction for expansion chambers than for loops.

3.4 1D Finite Element Method for Modal Analysis

Due to the dependence of each element transfer matrix on the excitation frequency, it is not possible to apply a conventional modal analysis procedure to extract the eigenvalues and eigenvectors of the acoustic problem. In OpenPulse, a simple 1D acoustic Finite Element is implemented considering the same mesh used for FETM. The following acoustic mass and stiffness element matrices are considered, respectively [3]:

$$\mathbf{M}_e = \frac{S_f}{\rho_f l_e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \tag{61}$$

and

$$\mathbf{K}_e = \frac{S_f l_e}{6\rho_f c_f^2} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \tag{62}$$

where l_e is the element length. All element matrices are assembled into global acoustic mass and stiffness matrices, and the modal analysis can be performed as discussed in Theory Reference E (to be written).

4 Nomenclature

- ALC = Acoustic length correction, m
- TMM = Transfer matrix method
- TPM = Two-port model/s
- MMM = Mobility matrix method
- SMM = Stifness matrix method
- TM = Transfer matrix
- MM = Mobility matrix
- SM = Stiffness matrix
- BC = Boundary condition
- TL = Transmission loss
- BC's = Boundary conditions
- EF = Equivalent fluid
- FEM = Finite element method
- FETM = Finite element transfer method
- AIV = Acoustic-induced vibration
- FIV = Flow-induced vibration
- FIVN = Flow-induced noise and vibration
- LHS = Left hand side
- RHS = right hand side
- DOF = degree of freedom
- DOF's = degrees of freedom
- LRF = Low reduced frequency
- PDES = Partial differential equations
- Wo = Womersley number, [-]
- Re = Womersley number, [-]
- St = Strouhal number, [-]
- Stk = Stokes number, [-]
- He = Helmholtz number, [-]
- Pr = Prandtl number, [-]

- 1D = one-dimensional
- 2D = two-dimensional
- 3D = three-dimensional
- J_0 = first kind Bessel function of order 0
- J_1 = first kind Bessel function of order 1
- J_2 = first kind Bessel function of order 2
- H_1 = Struve function of order 1
- p = nodal pressure, Pa
- q = inner nodal volume velocity, m^3/s
- u = particle velocity , m/s
- c = speed of sound, m/s
- $f = \omega/2\pi$ = frequency, Hz
- S = internal area of the pipe‘s cross section, m^2
- r = pipe radius, m
- z = specific acoustic impedance, Pas/m
- z_r = specific acoustic radiation impedance, Pas/m
- Z = acoustic impedance, Pas/m^3
- Z_r = acoustic radiation impedance, Pas/m^3
- Z_m = mechanical impedance, Ns/m

4.1 Greek Symbols

- ρ = density, kg/m^3
- ω = rotational frequency, rad/s
- δ = end correction function, dimensionless

4.2 Subscripts

- A = related to acoustic
- 0 = related to undisturbed fluid medium properties
- e = related to elemental
- f = related to fluid
- i = related to inner

- o = related to outer
- n= related to index of elements ($n = 1, 2, \ldots, N$)
- j= related to index of nodes ($j = 1, 2, \ldots, J$)
- ij= related to matrix entries index ($i, j = 1, 2, \ldots, I, J$)

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